REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER DTNSRDC-78/061	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) GOVERNING EQUATIONS FOR VISCOUS COMPRESSIBLE FLOW		5. TYPE OF REPORT & PERIOD COVERED Final rept. Apr - Jun 78
		6. PERFORMING ORG. REPORT NUMBER Aero Report-1249
7. AUTHOR(s) William J.H. Smithey		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS David W. Taylor Naval Ship R&D Center Bethesda, Maryland 20084		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 62241N Work Unit 1-1600-078
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Air Systems Command AIR-320D Washington, D.C. 20361		12. REPORT DATE August 1978
		13. NUMBER OF PAGES 25
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DDC
RECEIVED
AUG 30 1978
B

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Viscous Flow Equations   Thermal Conductivity
Continuity Equation
Momentum Equation
Energy Equation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Starting with the frictionless flow equations, governing equations for viscous compressible flow are developed from a physical molecular transport model by forming a first perturbation of the average flow property value transported across a surface element by two distinct groups of molecules. This results in a continuity equation with viscous correction terms.

DD ₁ FORM JAN 73 1473   EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

387695   78 08 29 042

TABLE OF CONTENTS

iii

NOTATION

a	One half the average molecular speed
$\bar{c}$	Velocity vector of molecules relative to $\bar{U}$
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
e	Internal energy per unit mass
F	Body force per unit mass potential
$\bar{I}$	Idem factor, $\bar{I} = ii + jj + kk$
$\bar{n}$	Outward drawn unit vector normal to the surface element dS
p	Static pressure
$\bar{Q}$	Energy flux vector
q	Heat addition rate per unit mass
R	Universal gas constant divided by molecular weight
S	Closed surface bounding V
SQR( )	Square root of the quantity in parenthesis
$\bar{t}$	Vector tangent to the surface element dS
T	Absolute static temperature
$\bar{U}$	Flow velocity vector
V	Arbitrary volume
X	Flow property value at the surface element associated with those molecules going out of V
Y	Flow property value at the surface element associated with those molecules going into V
λ	Average distance of molecular collisions from the surface element, two-thirds of the mean free path
μ	Viscosity
π	3.14159

ρ   Density, mass per unit volume

τ   Average transit time of molecules from collision
             point to the surface element, $\tau = \lambda/a$

Special Symbols

$\partial(\ )$ Partial time derivative of quantity in parenthesis

$(\ )'$ Property of molecules moving out of V through
             the surface element

$(\ )''$ Property of molecules moving into V through
             the surface element

$(\ )_c$ Conjugate of $(\ )$

ABSTRACT

Starting with the frictionless flow
equations, governing equations for viscous
compressible flow are developed from a phys-
ical molecular transport model by forming a
first perturbation of the average flow prop-
erty value transported across a surface ele-
ment by two distinct groups of molecules.
This results in a continuity equation with
viscous correction terms.

ADMINISTRATIVE INFORMATION

INTRODUCTION

The frictionless flow equations may be written in terms
of a volume integral equal to a surface integral, where the
surface integral integrand accounts for flow of some physical
property across a surface element. The molecules crossing the
surface element fall into two groups, those with a component
of relative velocity in the same direction as the surface
element unit vector $\bar{n}$ and those with a component opposite to
$\bar{n}$. By applying the simple physical molecular model in which
some average flow property is transported from the last aver-
age collision point across the surface element to these two
distinct groups of molecules, correction terms are obtained
for the frictionless flow continuity equation as well as

1

correction terms for the momentum and energy equations. The correction terms for the continuity equation are found to be essential for the solution of certain physical problems.

DEVELOPMENT

The frictionless flow equations may be written using dyadic and vector notation in terms of volume and surface integrals[1]*

$$\int \partial(\rho)dV = - \int \bar{n} \cdot (\rho\bar{U})dS \tag{1}$$

$$\int [\partial(\rho\bar{U}) - \rho\nabla F]dV = - \int \bar{n} \cdot (\rho\bar{U}\bar{U} + p\bar{n}\bar{n})dS \tag{2}$$

$$\int [\partial(\rho e + \rho\bar{U} \cdot \bar{U}/2 - \rho F) - \rho q]dV$$

$$= - \int \bar{n} \cdot (\rho e + \rho\bar{U} \cdot \bar{U}/2 - \rho F + p)\bar{U}dS \tag{3}$$

where $\partial(\ )$ denotes the partial derivative with respect to time, ∇F is the body force per unit mass due to the potential F, and -F is the potential energy per unit mass. These equations neglect molecular transport effects so there is no viscosity or thermal conductivity. Thus, in a volume element containing the surface element $\bar{n}dS$, the molecular velocity distribution is locally Maxwellian. Then, half of the

---

*A complete listing of references is given on page 17.

2

molecules with density ρ' will have velocity $\overline{c}'$ relative to $\overline{U}$ so that

$$\overline{n} \cdot \overline{c}' \geqslant 0 \qquad (4)$$

and the remaining molecules with density ρ'' will have

$$\overline{n} \cdot \overline{c}'' < 0 \qquad (5)$$

Then for Equation (1)

$$\rho\overline{U} = \rho'(\overline{U}+a\overline{n}) + \rho''(\overline{U}-a\overline{n}) \qquad (6)$$

identifies the two groups of molecules crossing $\overline{n}dS$, ρ' from inside V going out and ρ'' from outside V going in, where

$$\rho' = \rho'' = \rho/2 \qquad (7)$$

and

$$a = SQR(2RT/\pi) \qquad (8)$$

In Equation (8), a is found to be one-half the average molecular speed from kinetic gas theory,[2] R is the universal gas constant divided by the molecular weight, and T is the absolute static temperature.

Using $p = \rho RT$ in Equation (2)

$$(\rho\overline{U}\,\overline{U}+p\overline{n}\,\overline{n}) = (\rho\overline{U}\,\overline{U}+\rho RT\overline{n}\,\overline{n})$$
$$= [\rho'(\overline{U}+a\overline{n})(\overline{U}+a\overline{n})+\rho''(\overline{U}-a\overline{n})(\overline{U}-a\overline{n})] \qquad (9a)$$

identifies the two groups of molecules crossing $\overline{n}dS$ and the velocity associated with each group, where it is understood

3

that ρaa must be multiplied by $\pi/2$ when it occurs to
account for root mean square averaging and obtain the ρRT
term. Expanding Equation (9a) and using the idem factor[3] $\overline{I}$
to replace $\overline{n}\overline{n}$, because $\overline{n}\cdot\overline{n}\overline{n} = \overline{n}\cdot\overline{I} = \overline{n}$, one finds

$$(\rho\overline{U}\overline{U}+p\overline{n}\overline{n}) = \rho'(\overline{U}\overline{U}+a\overline{n}\overline{U}+a\overline{U}\overline{n}+RT\overline{I})$$
$$+ \rho''(\overline{U}\overline{U}-a\overline{n}\overline{U}-a\overline{U}\overline{n}+RT\overline{I}) \tag{9b}$$

Using $p = \rho RT$, $e = c_v T$, and $c_p = R + c_v$ in Equation (3)

$$\rho(c_p T+\overline{U}\cdot\overline{U}/2-F)\overline{U} = \rho'(c_p T+\overline{U}\cdot\overline{U}/2-F)(\overline{U}+a\overline{n})$$
$$+ \rho''(c_p T+\overline{U}\cdot\overline{U}/2-F)(\overline{U}-a\overline{n}) \tag{10}$$

identifies the two groups of molecules crossing $\overline{n}$dS and the
energy associated with each group.

To form the perturbation, assume some average flow prop-
erty value, e.g., X', is acquired by the ρ' molecules at their
last average collision point before crossing $\overline{n}$dS.
Then, if X is the average flow property at the surface element

$$X' = X - (\lambda/a)(\overline{U}+a\overline{n})\cdot\nabla(X) \tag{11a}$$

is the average flow property value acquired by the ρ' mole-
cules and transported across the surface element, where λ
is the average distance of the collision point from dS when
$\overline{n}\cdot\overline{U} = 0$. Also, λ is two-thirds of the mean free path. The
ratio (λ/a) is an average characteristic time, e.g., τ,

4

between molecular collisions and passage of the molecules through dS. Thus, Equation (11a) may also be written

$$X' = X - \tau(\overline{U}+a\overline{n}) \cdot \nabla X \tag{11b}$$

Similarly, if Y'' is an average flow property value of the ρ'' molecules

$$Y'' = Y + \tau(a\overline{n}-\overline{U}) \cdot \nabla Y \tag{12}$$

The time averaging of molecular motion in a volume element provides the basis for the perturbation model and this process should be examined briefly. At standard conditions there are approximately 2.7×10^{19} molecules in a cubic centimeter of gas. The dimension of λ is approximately 4.3×10^{-6} cm and a is approximately 2.2×10^4 cm/s, hence, $\tau = 2 \times 10^{-10}$ seconds. Then, as the molecules enter and leave a cubic volume element of dimension 2λ, there will be approximately $6 \times 8.6 \times 8.6 \times 2.2 \times 2.7 \times 10^{11} = 2.6 \times 10^{14}$ discrete values of a particular frictionless flow property ($\rho, \overline{U}, p$, etc.) per second. If the macroscopic time element associated with the flow properties in the frictionless flow equations is taken as one microsecond, then during that time element there will be approximately 2.6×10^8 discrete values of the frictionless flow property that form the average value associated with the location of the cubic volume element at that particular time. During this microsecond, approximately 2.2×10^7 molecules will enter a side of the cubic volume element and

5

penetrate to its center; thus transporting their flow property value across a surface element at the center of the volume element. The flow property value transported across the surface element by these penetrating molecules may be different from the average frictionless flow property value of the volume element, which is macroscopically flowing through the surface element. The perturbation model attempts to account for this difference. To be consistent, the perturbation model will be applied to the surface element flux of all three frictionless flow equations.

Substituting Equations (6), (11b), and (12) in Equation (1) forms the continuity perturbation,

$$\int \partial(\rho)dV = - \int \bar{n} \cdot [\rho'(\bar{U}+a\bar{n})+\rho''(\bar{U}-a\bar{n})$$

$$-\tau(\bar{U}+a\bar{n}) \cdot \nabla(\rho'\bar{U}+\rho'a\bar{n})+\tau(a\bar{n}-\bar{U}) \cdot \nabla(\rho''\bar{U}-\rho''a\bar{n})]dS$$

$$= - \int \bar{n} \cdot [\rho\bar{U}-\tau\bar{U} \cdot \nabla(\rho\bar{U})-\tau a\bar{n} \cdot \nabla(\rho a\bar{n})]dS \qquad (13a)$$

where $X = \rho'(\bar{U}+a\bar{n})$ and $Y = \rho''(\bar{U}-a\bar{n})$ has been used.

Expanding the last term of Equation (13a) one obtains a component $\bar{n} \cdot (\bar{n} \cdot \nabla\bar{n})$. The derivative $\bar{n} \cdot \nabla\bar{n}$ will be a vector $\bar{t}$ which is tangent to dS. Thus, $\bar{n} \cdot (\bar{n} \cdot \nabla\bar{n}) = \bar{n} \cdot \bar{t} = 0$. With this and $\bar{n} \cdot \bar{n} = 1$, Equation (13a) may be rewritten

$$\int \partial(\rho)dV = - \int \bar{n} \cdot [\rho\bar{U}-\tau\bar{U} \cdot \nabla(\rho\bar{U})-\lambda\nabla(\rho a)]dS \qquad (13b)$$

The last two terms of Equation (13b), dotted by $\bar{n}$, represent
a decrease of the frictionless flow mass flux $\bar{n} \cdot (\rho \bar{U})$ crossing
dS due both to a shift of the last two average collision points
by the frictionless flow velocity and to a molecular mass
flux gradient. In the last term of Equation (13b), the λ may
be replaced by viscosity, $\mu = \rho a \lambda$. With this and the diver-
gence theorem, Equation (13b) becomes

$$\int \{\partial(\rho) + \nabla \cdot (\rho \bar{U}) - \nabla \cdot [\tau \bar{U} \cdot \nabla(\rho \bar{U}) + \mu \nabla \ln(\rho a)]\} dV = 0 \qquad (14)$$

Because the volume is arbitrary, the integrand of Equation
(14) must be zero. Thus, the continuity equation becomes

$$\partial(\rho) + \nabla \cdot (\rho \bar{U}) = \nabla \cdot [\tau \bar{U} \cdot \nabla(\rho \bar{U}) + \mu \nabla \ln(\rho a)] \qquad (15)$$

where the terms on the right provide corrections to the
macroscopic frictionless flow continuity equation due both to
a shift of the last two average collision surfaces, caused by
the flow velocity, and to the molecular mass transport
gradient.

Substitute Equations (9a), (9b), (11b) and (12) in Equa-
tion (2) to form the momentum perturbation

7

$$\int [\partial(\rho\bar{U}) - \rho\nabla F]dV$$

$$= - \int \bar{n}\cdot\{\rho'(\bar{U}+a\bar{n})(\bar{U}+a\bar{n}) + \rho''(\bar{U}-a\bar{n})(\bar{U}-a\hat{n})$$

$$-\tau(\bar{U}+a\bar{n})\cdot\nabla[\rho'(\bar{U}+a\bar{n})(\bar{U}+a\bar{n})]$$

$$+\tau(a\bar{n}-\bar{U})\cdot\nabla[\rho''(\bar{U}-a\bar{n})(\bar{U}-a\bar{n})]\}dS$$

$$= - \int \bar{n}\cdot[\rho\bar{U}\bar{U}+p\bar{I}-\tau\bar{U}\cdot\nabla(\rho\bar{U}\bar{U}+p\bar{I})$$

$$-\lambda\bar{n}\cdot\nabla(\rho a\bar{n}\bar{U}+\rho a\bar{U}\bar{n})]dS \qquad (16)$$

Using the divergence theorem

$$\int [\partial(\rho\bar{U})+\nabla\cdot(\rho\bar{U}\bar{U})+\nabla p-\rho\nabla F-\nabla\cdot[\tau\bar{U}\cdot\nabla(\rho\bar{U}\bar{U})]$$

$$-\nabla(\tau\bar{U}\cdot\nabla p)]dV = \int \bar{n}\cdot[\lambda\bar{n}\cdot\nabla(\rho a\bar{n}\bar{U}+\rho a\bar{U}\bar{n})]dS \qquad (17)$$

The surface integral in Equation (17) contains the symmetric dyadic $(\rho a\bar{n}\bar{U}+\rho a\bar{U}\bar{n})$ whose value depends on the orientation of $\bar{n}$. Carrying out the differentiation for $\bar{n}\cdot\nabla$, the surface integral integrand becomes

$$\bar{n}\cdot[\lambda\bar{n}\cdot\nabla(\rho a\bar{n}\bar{U}+\rho a\bar{U}\bar{n})]$$

$$= \bar{n}\cdot\lambda\{(\bar{n}[\bar{n}\cdot\nabla(\rho a\bar{U})]+\bar{t}(\rho a\bar{U})$$

$$+[\bar{n}\cdot\nabla(\rho a\bar{U})]\bar{n}+(\rho a\bar{U})\bar{t}\} \qquad (18)$$

8

The dyadic on the right side of Equation (18) is also symmetric. Using $\bar{n}\cdot\bar{n} = 1$ and $\bar{n}\cdot\bar{t} = 0$, this may be rewritten

$$\bar{n}\cdot[\lambda\bar{n}\cdot\nabla(\rho a\bar{n}\bar{U}+\rho a\bar{U}\bar{n})]$$

$$= \bar{n}\cdot\lambda[\nabla(\rho a\bar{U})+[\bar{n}\cdot\nabla(\rho a\bar{U})\bar{n}+(\rho a\bar{U})\bar{t}]\qquad(19)$$

The dyadic on the right side of Equation (19) must be symmetric to preserve equilibrium of the volume element,[4] thus the last terms must be replaced by $[\nabla(\rho a\bar{U})]_c$, which is the conjugate of $\nabla(\rho a\bar{U})$. The normal component $\bar{n}\cdot[\bar{n}\cdot\nabla(\rho a\bar{U})]$ is found to be equal to the normal component $\bar{n}\cdot(\bar{n}\cdot[\nabla(\rho a\bar{U})]_c)$ by merely expanding; thus it is the tangential component of $\bar{n}\cdot[\nabla(\rho a\bar{U})]_c$ which must be equal to $\bar{n}\cdot(\rho a\bar{U})\bar{t}$ to preserve the equilibrium of the volume element. Other authors[5] showed the requirement for a symmetric dyadic using the principle of angular momentum conservation, so that this view could be used instead of preserving equilibrium of the volume element. Perhaps the opposite point of view would be to assume that a volume element is not in equilibrium and has started to spin up. Then, in a microsecond, the molecules within the volume element would flux back and forth about a thousand times, transporting tangential momentum which opposes the spin to reestablish the volume element's equilibrium.

9

In any case, the dyadic must be symmetric. With Equation (19) and the divergence theorem, Equation (17) becomes

$$\int \{\partial(\rho\overline{U}) + \nabla\cdot(\rho\overline{U}\,\overline{U}) + \nabla p - \rho\nabla F - \nabla\cdot[\tau\overline{U}\cdot\nabla(\rho\overline{U}\,\overline{U})]$$

$$-\nabla(\tau\overline{U}\cdot\nabla p) - \nabla\cdot(\lambda\nabla(\rho a\overline{U}) + \lambda[\nabla(\rho a\overline{U})]_c)\}dV = 0 \qquad (20)$$

Because the volume is arbitrary, the integrand of Equation (20) must be zero. Using μ, the momentum equation becomes

$$\partial(\rho\overline{U}) + \nabla\cdot(\rho\overline{U}\,\overline{U}) + \nabla p - \rho\nabla F = \nabla\cdot[\tau\overline{U}\cdot\nabla(\rho\overline{U}\,\overline{U})]$$

$$+ \nabla(\tau\overline{U}\cdot\nabla p) + \nabla\cdot\left[(\mu/\rho a)\{\nabla(\rho a\overline{U}) + [\nabla(\rho a\overline{U})]_c\}\right] \qquad (21)$$

where the first two terms on the right, involving τ, provide a correction due to a shift of the last average collision surfaces by the flow velocity. The last terms provide a correction due to the molecular momentum transport gradient. The dyadic $\nabla(\rho a\overline{U})$ and its conjugate may be expanded to show the correction that is due strictly to viscosity and velocity gradient

$$(\mu/\rho a)\{\nabla(\rho a\overline{U}) + [\nabla(\rho a\overline{U})]_c\}$$

$$= \mu[\nabla\overline{U} + (\nabla\overline{U})_c] + \mu[(\nabla\ln\rho a)\overline{U} + \overline{U}(\nabla\ln\rho a)] \qquad (22)$$

10

Using Equations (10), (11b), and (12) in Equation (3) to form the energy perturbation

$$\int \{\partial(\rho e + \rho\overline{U}\cdot\overline{U}/2 - \rho F) + \nabla\cdot[(\rho e + \rho\overline{U}\cdot\overline{U}/2 - \rho F + p)\overline{U}] - \rho q\}dV$$

$$= \int \overline{n}\cdot\{\tau(\overline{U}+a\overline{n})\cdot\nabla[\rho'(c_p T + \overline{U}\cdot\overline{U}/2 - F)(\overline{U}+a\overline{n})]$$

$$+ \tau(\overline{U}-a\overline{n})\cdot\nabla[\rho''(c_p T + \overline{U}\cdot\overline{U}/2 - F)(\overline{U}-a\overline{n})]\}dS$$

$$= \int \overline{n}\cdot\{\tau\overline{U}\cdot\nabla[\rho(c_p T + \overline{U}\cdot\overline{U}/2 - F)\overline{U}]$$

$$+ \lambda\nabla[\rho a(c_p T + \overline{U}\cdot\overline{U}/2 - F)]\}dS \tag{23}$$

With the divergence theorem, Equation (23) may be written as a volume integral equal to zero, and, since the volume is arbitrary, the integrand is zero. Then, using $\lambda = \mu/(\rho a)$, the energy equation may be written

$$\partial(\rho[c_v T + \overline{U}\cdot\overline{U}/2 - F]) + \nabla\cdot[\rho(c_p T + \overline{U}\cdot\overline{U}/2 - F)\overline{U}] - \rho q$$

$$= \nabla\cdot\{\tau\overline{U}\cdot\nabla[\rho(c_p T + \overline{U}\cdot\overline{U}/2 - F)\overline{U}] + \mu\nabla(c_p T + \overline{U}\cdot\overline{U}/2 - F)$$

$$+ \mu(c_p T + \overline{U}\cdot\overline{U}/2 - F)\nabla[\ln(\rho a)]\} \tag{24}$$

DISCUSSION

The significant result obtained from this development is the continuity equation with molecular transport correction terms, Equation (15). These terms correct the macroscopic frictionless flow continuity equation and are the same order of magnitude as the correction terms for the momentum

11

and energy equations. It should also be noted that the
momentum and energy equations, given by Equations (21) and
(24), provide new correction terms due to a shift of the
molecular collision surfaces, on both sides of the surface
element dS, by the flow velocity. When viewed in the surface
integrals, all of these correction terms may be physically
interpreted as perturbations to the locally Maxwellian mole-
cular velocity distribution function. The perturbations re-
sult from the collision surfaces being situated on both sides
of the surface element dS, rather than coincident with it.

Two simple examples will illustrate the nature of these
correction terms. First, consider a thermally insulated
container of gas at rest, with uniform pressure, without body
forces, and with a thermally insulated partition so that the
two compartments of the container are at different tempera-
tures. Now, if the partition is removed, there will be un-
steady effects due to molecular motion which initially trans-
port mass and energy across the surface elements where the
partition was located. For the two compartments, let $T_2 < T_1$,
and because the pressures are equal $\rho_1 < \rho_2$. The net mass
flux from 1 to 2 will be

$$(\rho_1 a_1 - \rho_2 a_2)/2$$

$$= [P/SQR(2R\pi)][1/SQR(T_1) - 1/SQR(T_2)] \qquad (25)$$

12

The net energy flux from 1 to 2 will be

$$(\rho_1 a_1 c_p T_1 - \rho_2 a_2 c_p T_2)/2$$

$$= [pc_p/SQR(2R\pi)][SQR(T_1)-SQR(T_2)] \qquad (26)$$

Thus, near the surface element there will be a time rate of change of density $\partial(\rho)$ even though the macroscopic flow velocity is zero. This effect is included in Equation (15). Since these two net effects occur at the molecular level within a few mean free paths, a net pressure gradient will be established across the surface element which starts to drive the momentum equation. Of course the system without the partition will eventually reach a new state of thermodynamic equilibrium.

As a second example, consider the flow near the center of a long thermally insulated pipe filled with gas, without body forces, which is conducting heat in a steady state condition. Taking Equation (15) back to a surface integral, the macroscopic mass flux will be

$$\rho\overline{U} = \tau\overline{U}\cdot\nabla(\rho\overline{U}) + \mu\nabla\ln(\rho a) \qquad (27)$$

which is exactly balanced by the small microscopic mass flux in the -U direction.

Taking Equation (24) back to a surface integral, the energy
flux vector $\bar{Q}$ will be

$$\bar{Q} = \rho(c_pT+\bar{U}\cdot\bar{U}/2)\bar{U} - \tau\bar{U}\cdot\nabla[\rho(c_pT+\bar{U}\cdot\bar{U}/2)\bar{U}]$$

$$- \mu\nabla(c_pT+\bar{U}\cdot\bar{U}/2) - \mu(c_pT+\bar{U}\cdot\bar{U}/2)\nabla\ln(\rho a) \qquad (28)$$

This equation shows that the energy is transported macro-
scopically by $\rho\bar{U}$ as well as microscopically by the molecular
activity. Using Equation (27), the energy flux vector may
be written

$$\bar{Q} = - [\tau\bar{U}\cdot\nabla(\rho\bar{U})+\mu\nabla\ln(\rho a)][\tau\bar{U}\cdot\nabla(c_pT+\bar{U}\cdot\bar{U}/2)]$$

$$- \mu\nabla(c_pT+\bar{U}\cdot\bar{U}/2) \qquad (29)$$

The principal term contributing to the energy flux vector in
Equation (29) is $-\mu\nabla(c_pT)$. If the body force potential F
were not zero, the energy flux vector $\bar{Q}$ would include this
effect from Equation (24). Thus, if a long pipe were used
to experimentally determine the coefficient of thermal con-
ductivity, the result would be affected slightly by the
orientation of the pipe to the potential gradient. In this
case, the approximate net energy flux would be

$$\bar{n}\cdot\bar{Q} = -\mu\bar{n}\cdot\nabla(c_pT-F) \qquad (30)$$

The correction terms involving $\tau\bar{U}$ in Equations (15),
(21), and (24) will become significant in high-speed flows,

such as found in ballistic reentry and shock wave calculations. This occurs when $\bar{U}$ approaches the same order of magnitude or is greater than a, so that $\tau\bar{U}$ is several orders of magnitude larger than in the two examples given above.

CONCLUSIONS

The governing equations for viscous compressible flow have been derived by using a simple molecular transport model to form an approximate perturbation of the frictionless flow equation. The molecular transport correction terms in the continuity equation are of the same order of magnitude as the correction terms in the momentum and energy equations. It appears that all of these correction terms should be considered initially for the proper analysis of diverse flow processes.

REFERENCES

1. Liepmann, H.W. and A. Roshko, "Elements of Gas-dynamics," John Wiley & Sons, Inc., New York (1957), pp. 180-188.

2. Sears, F.W., "An Introduction to Thermodynamics, the Kinetic Theory of Gases, and Statistical Mechanics," Second Edition, Addison-Wesley Publishing Co., Inc., Reading, Mass. (1959), pp. 261-269.

3. Vavra, M.H., "Aero-Thermodynamics and Flow in Turbomachines," John Wiley & Sons, Inc., New York (1960), p. 538.

4. Schlichting, H., "Boundary Layer Theory," Sixth Edition, McGraw-Hill Book Co., Inc., New York (1968), p. 47.

5. Phillips, H.B., "Vector Analysis," Seventh Printing, John Wiley & Sons, Inc., New York (1946), p. 230.

INITIAL DISTRIBUTION

Copies

3 CHONR
   1 430B
   1 432
   1 438

1 NRL

1 NADC

1 USNA

3 NAVPGSCOL
   1 Prof. Fuhs
   1 Prof. Netzer
   1 Library

1 NWC

1 NSWC/White Oak

1 NSWC/Dahlgren

9 NAVAIRSYSCOM
   1 AIR 03C
   1 AIR 310
   3 AIR 320D
   4 AIR 5301

2 NAVSEASYSCOM
   1 Library
   1 SEA-03513 (Pasiuk)

12 DDC

1 AFOSR

1 AFFDL

2 USAFA
   1 LTCOL Gallington
   1 Library

1 AF INST TECH

1 AEDC

1 NASA, HQ

Copies

3 NASA Ames Research Center
   1 Library
   1 J.C. Biggers
   1 G. Vanderplaats

1 NASA Langley Research Center

1 NASA Lewis Research Center

1 National Science Foundation

1 U of Akron/Lib

2 Brown U/Div of Engr
   1 Prof. Kestin
   1 Library

1 Calif Inst of Tech
   Grad Aero Labs

1 Calif Inst of Tech
   Jet Propulsion Lab

1 U of Calif/Berkeley Lib

1 U of Calif/La Jolla Lib

1 Catholic U of America/Lib

1 U of Cincinnati/Lib

1 Clemson U/Lib

1 Cornell U/Lib

1 Georgia Inst Tech/Lib

1 Harvard U/Lib

1 Johns Hopkins U/Lib

1 U of Illinois/Lib

1 U of Maryland/Lib

1 U of Michigan/Lib

1 U of N Carolina/Lib

Copies

1   N Carolina State U,
     Raleigh Lib

1   Pennsylvania State U/Lib

1   Princeton U/Lib

1   Purdue U/Lib

1   Stanford U/Lib

1   U of Tenn Space Inst/Lib

1   U of Virginia/Lib

1   Virginia Polytech Inst/Lib

1   U oF Washington/Lib

1   W Virginia U/Lib

1   AIAA

1   ARO Inc/Lib

1   Boeing Company/Seattle

1   Calspan Corp/Buffalo

1   Douglas Aircraft Co/Lib

1   DYNATECH/Lib

1   Flow Research/Lib

1   Gen Dynamics Convair/Lib

1   Grumman Aerospace Corp/Lib

1   Inst for Defense Analyses

1   Lockheed-Georgia Co/Lib

1   Lockheed Missiles
     & Space/Lib

1   MIT/Lib

Copies

1   New York U/Courant Inst
     Math Sciences/Lib

1   Northrop Corp Labs/Lib

1   Rockwell International,
     Columbus/Lib

1   Rockwell International,
     Science Center
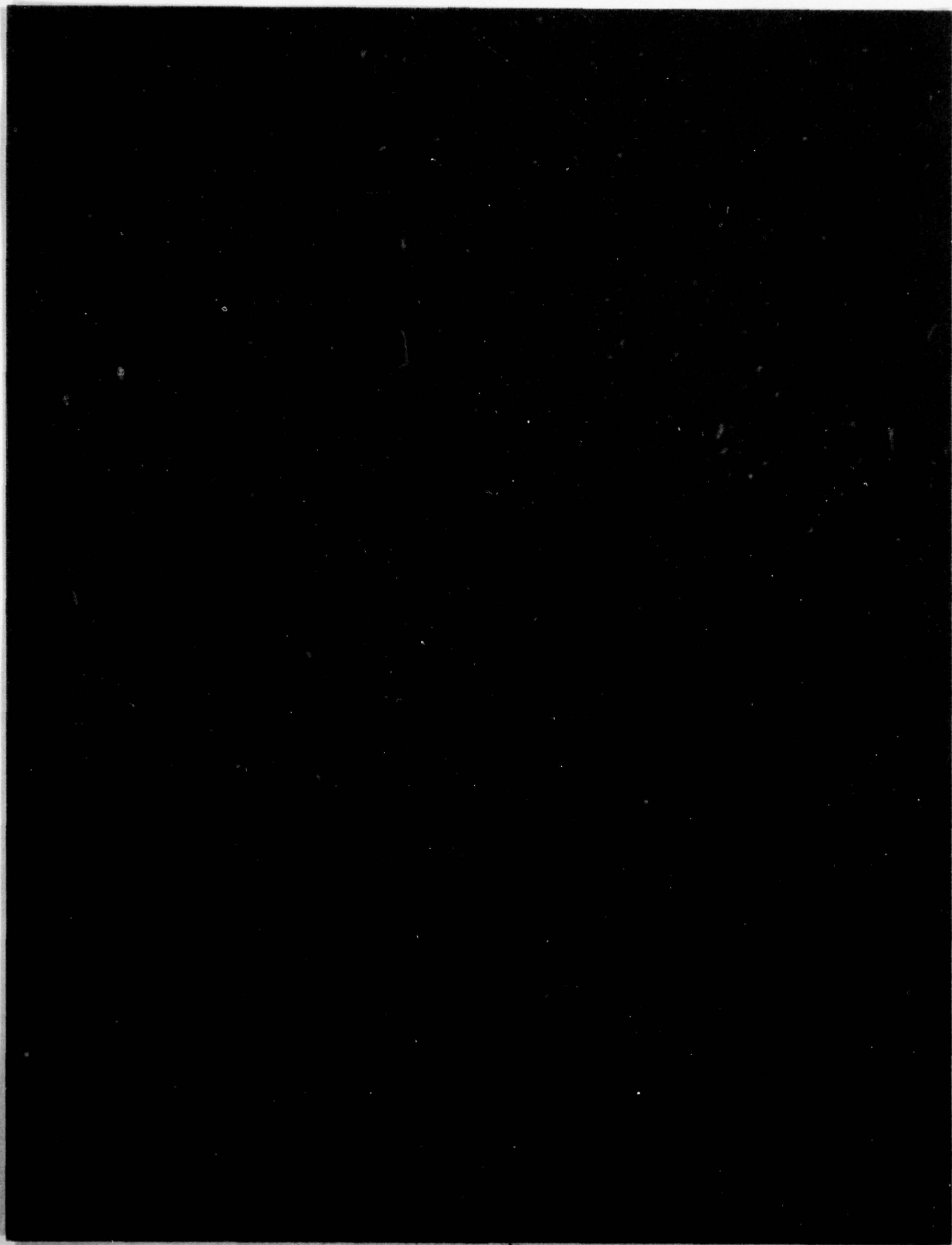
1   TRW Systems Group/Lib

CENTER DISTRIBUTION

Copies	Code	
10	5214.1	Reports Distribution
1	522.1	Library (C)
1	522.2	Library (A)
2	522.3	Aerodynamics Library

AD-A058 140   DAVID W TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CE--ETC F/G 20/4
               GOVERNING EQUATIONS FOR VISCOUS COMPRESSIBLE FLOW.(U)
               AUG 78   W J SMITHEY
UNCLASSIFIED   AERO-1249   DTNSRDC-78/061   NL

2 OF 2
AD
A058140

SUPPLEMENTARY

INFORMATION

SUPPLEME

INFORMA

ERRATA SHEET

DTNSRDC Report 78/061, August 1978
by William J.H. Smithey

GOVERNING EQUATIONS FOR VISCOUS COMPRESSIBLE FLOW

Please replace page 17 in subject report with attached page 17.

REFERENCES

1.   Liepmann, H.W. and A. Roshko, "Elements of Gas-
dynamics," John Wiley & Sons, Inc., New York (1957),
pp. 180-188.

2.   Sears, F.W., "An Introduction to Thermodynamics,
the Kinetic Theory of Gases, and Statistical Mechanics,"
Second Edition, Addison-Wesley Publishing Co., Inc.,
Reading, Mass. (1959), pp. 261-269.

3.   Vavra, M.H., "Aero-Thermodynamics and Flow in
Turbomachines," John Wiley & Sons, Inc., New York (1960),
p. 538.

4.   Schlichting, H., "Boundary Layer Theory," translated
by J. Restin, Fourth Edition, McGraw-Hill Book Co., Inc.,
New York (1960), p. 45; Also see Sixth Edition, McGraw-Hill
Book Co., Inc., New York (1968), p. 47.

5.   Phillips, H.B., "Vector Analysis," Seventh
Printing, John Wiley & Sons, Inc., New York (1946), p. 230.